Section 2.2

Limits: A Numerical and Graphical Approach

Link

Limits

One-Sided Limit

$$\lim_{x \to c^-} f(x) = L$$

The left limit of f is L if the y-values converge to L as x approaches c through values **less than** c.

$$\lim_{x \to c^+} f(x) = L$$

The right limit of f is L if the y-values converge to L as x approaches c through values **greater than** c.

Two-Sided Limit

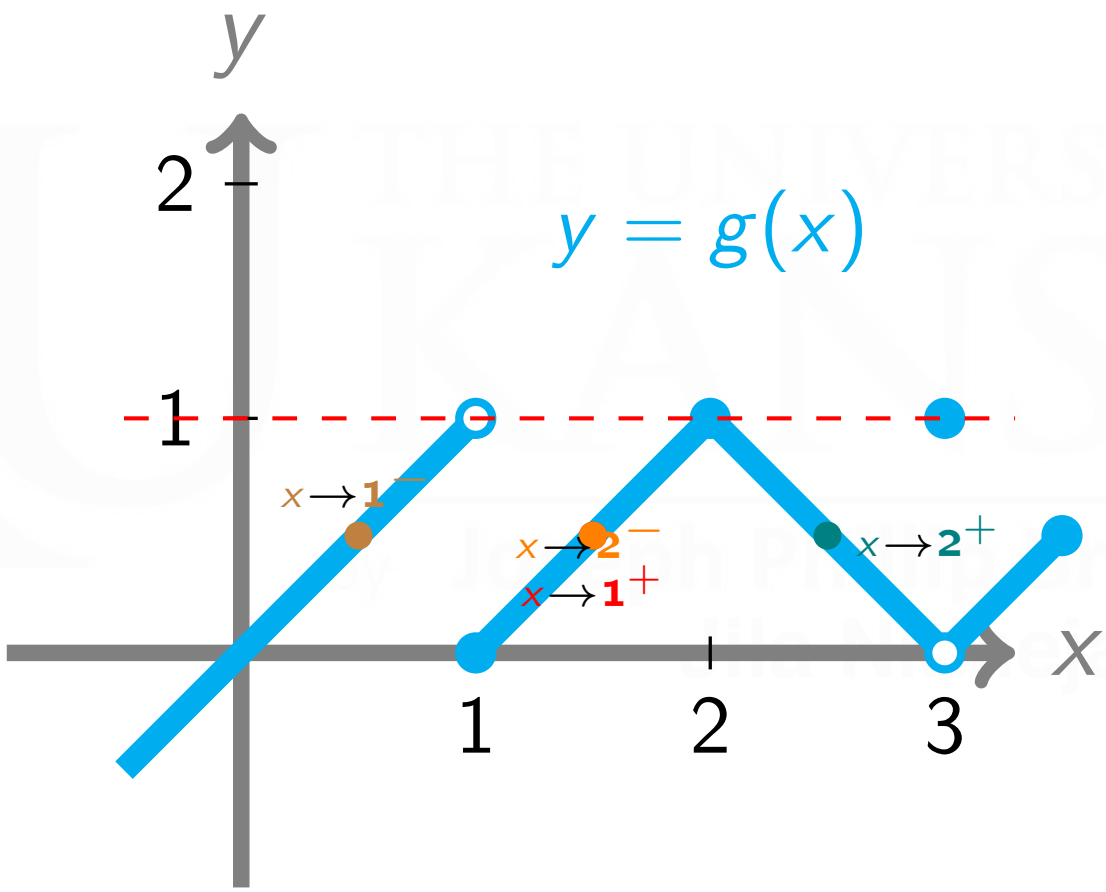
$$\lim_{x \to c} f(x) = L$$

The two-sided limit of f is L **ONLY IF** both one-sided limits equal L.

Link

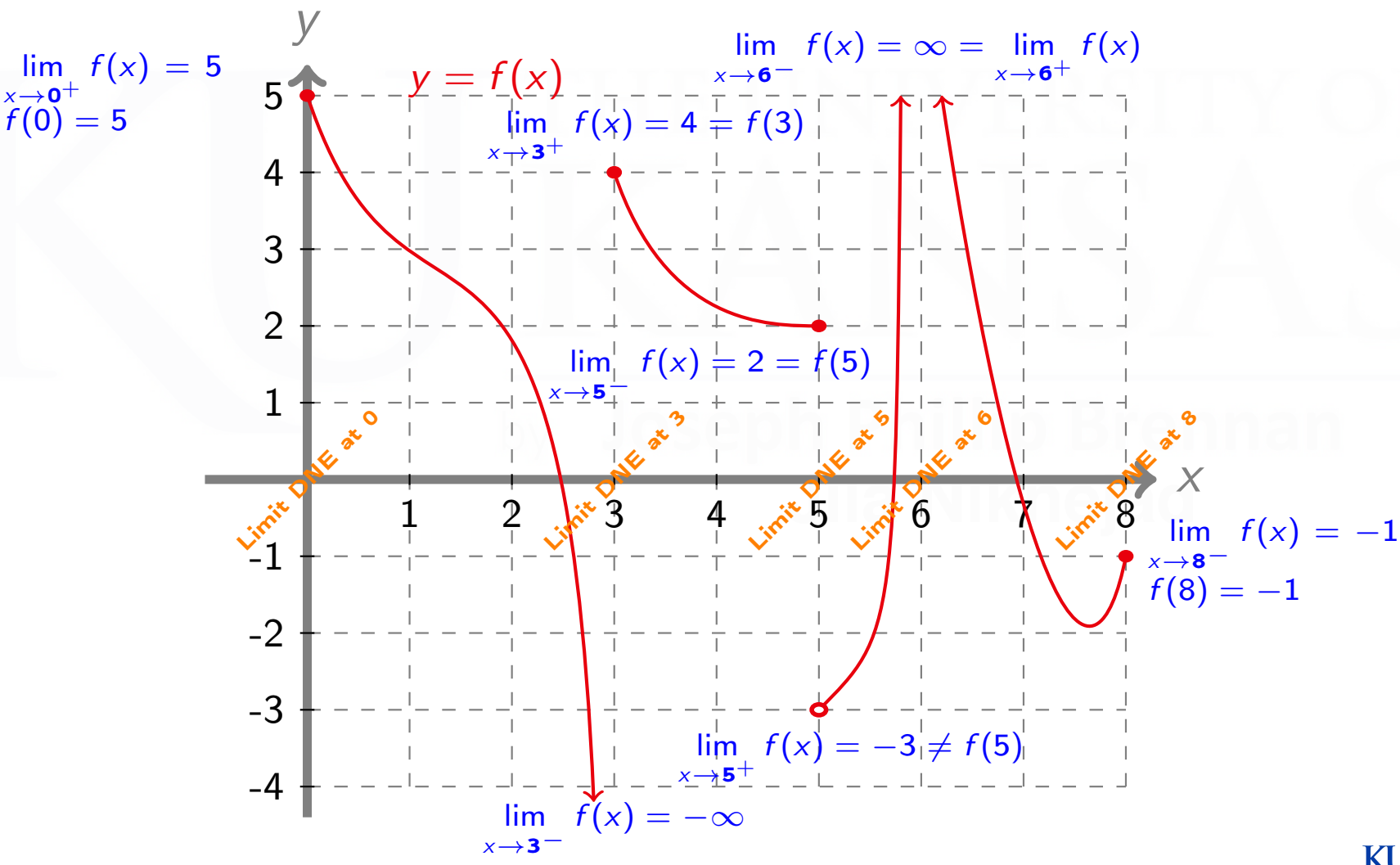
Identifying Limits on a Graph - Example I

Discuss the limits and actual values on the following graph:



Identifying Limits on a Graph - Example II

Discuss the limits and actual values on the following graph:

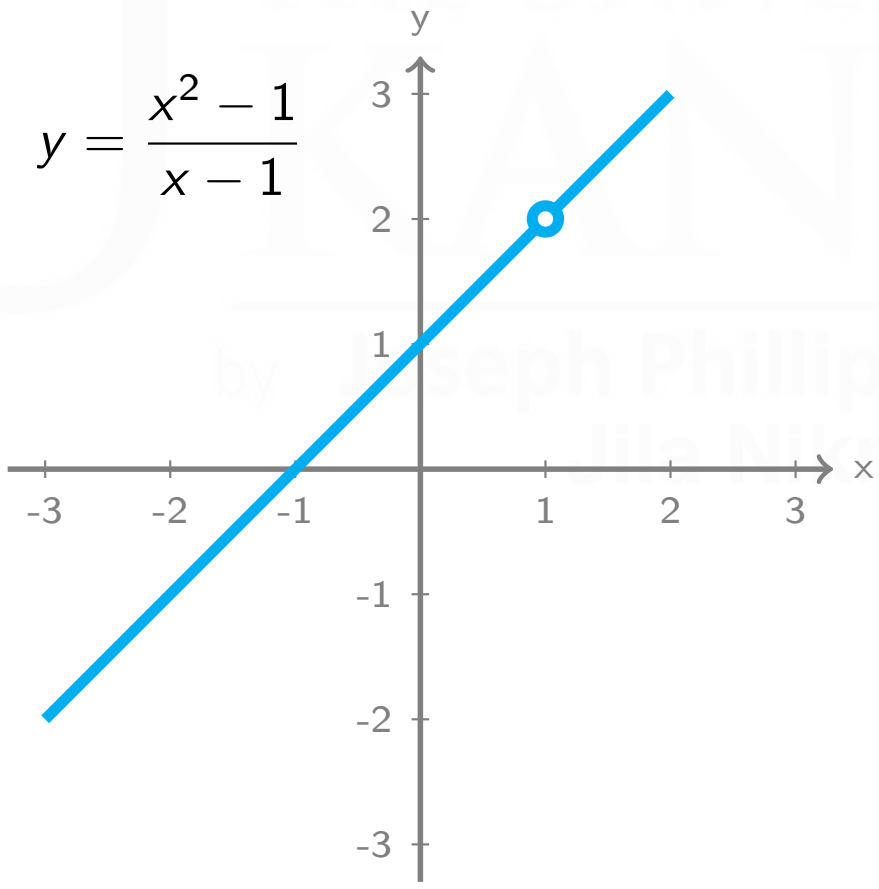


Estimating Limits

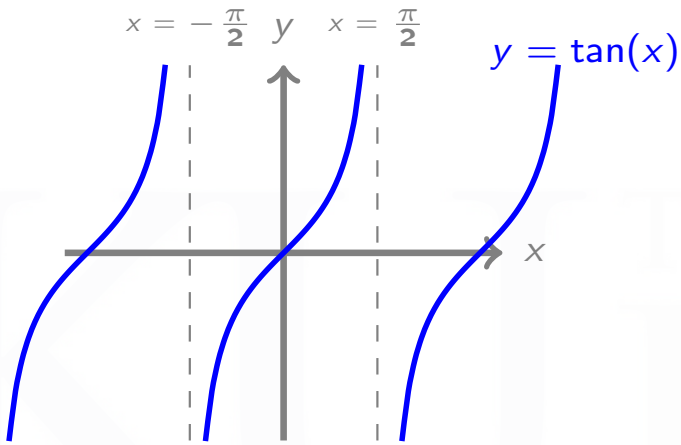
Let $f(x) = \dfrac{x^2 - 1}{x - 1}$. What is the limit of $f(x)$ as $x \to 1$?

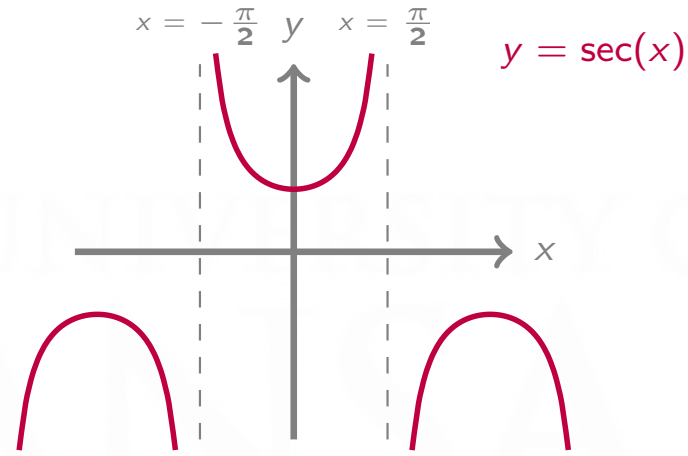
x	0.9	0.99	0.999	0.99999	1	1.000001	1.001	1.01	1.1
y	1.9	1.99	1.999	1.999999	DNE	2.0000001	2.001	2.01	2.1

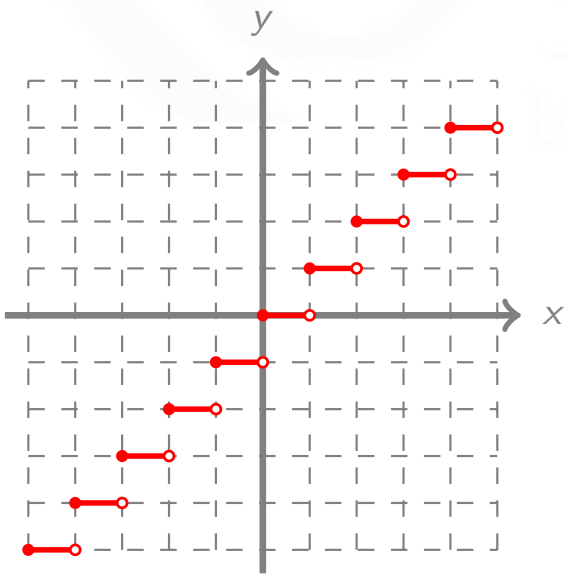
$$y = \frac{x^2 - 1}{x - 1}$$

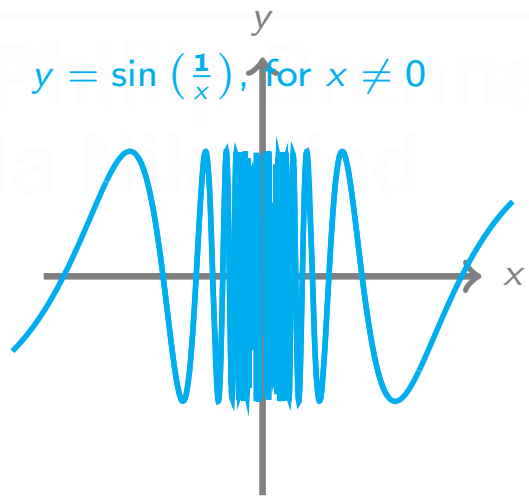


Limits That Don't Exist

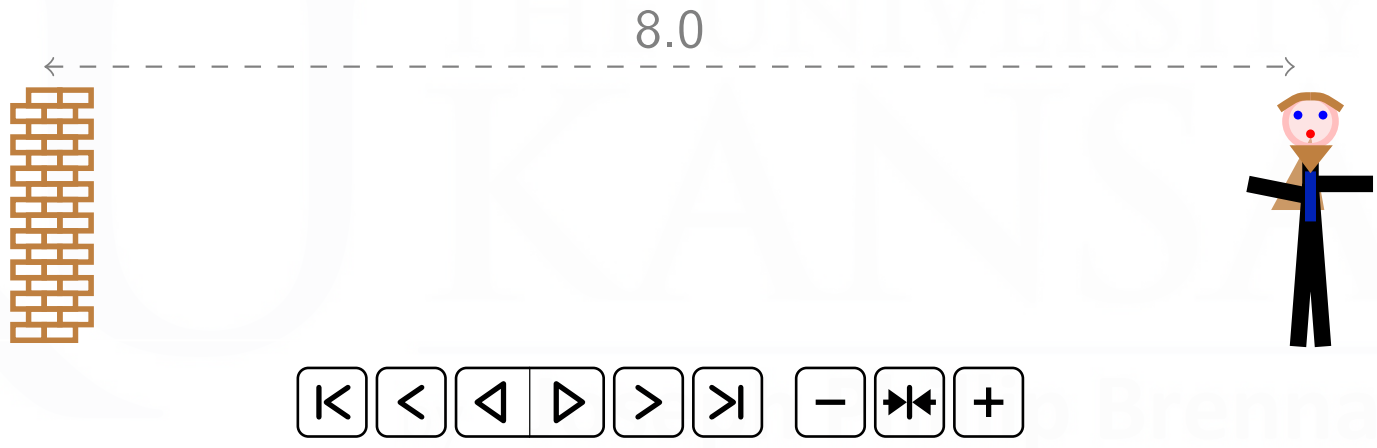








Zeno's Paradox



Zeno walks towards a wall by walking exactly half the distance remaining between him and the wall. Will he ever reach the wall?

The limit is the wall even if Zeno never reaches it!

Limits and Actual Values are INDEPENDENT of Each Other

We will be interested in certain situations where the limit and actual value at a point exist and are equal. This concept is called **continuity**; more in section 2.4!

- Hair Cuts: What are the one-sided limits at the instant of cutting?
- Meter Rates: Electric company increases charges and institutes fees at a certain point, what are the one sided limits at that point?

The Unexpected: Truman is sailing his boat triumphantly away from his home when he suddenly hits the transparent wall enclosing his world... only then does he understand that his world was artificial.

Niccol, A., & Weir, P. (1999). The Truman Show.

Sewer Hole

The limit exists if a path leads to a hole but the value of function is not the same as the limit.
Section 2.4 provides a definition to explain the above phenomena.